4 Electronics

Program Library

Networks
Circuits
Filters
Electrostatics
Electrodynamics
Radiation & Propagation

Sinclair Cambridge Programmable

Printed by Hobsons Press (Cambridge) Ltd

Electronics

4

CONTENTS

How to use these programs

Each program is arranged as follows:

1. On the left of the page, explanatory information and the 'execution sequence', the sequence of keystrokes necessary for running the program. Results displayed are printed in gold.
2. In the first column on the right hand side of the page, the sequence of keystrokes which make up the program.
3. In the second and third columns on the right hand side of the page, the program in check symbol and step number form (see section on checking the program).

Notes

1. Where a key has more than one function, the relevant function is printed as the keystroke in the first column

   e.g. the keystroke [8] (cos / arccos) may appear as 8, cos or arccos.
2. The symbol ▼ within a program always refers to the key [•/EE/–] (▼)
3. The symbol # refers to [3] (ChN/#)
4. The abbreviation gin is 'go if neg' and so refers to the key [1] (go if neg)

Entering the program

To enter a program into the calculator:

1. Press [▲▼] [▲▼] [2] (go to) [0] [0]	Display shows step programmed at 00 in check symbol form as described below.
2. Press [▲▼] [RUN] (learn)	No change in display.
3. Press the sequence of keys for the program as shown in the first column of the program page.	At each stage the step about to be overwritten is displayed. When the machine is first switched on every step is zero.
4. Press [C/CE]	Normal number display is resumed.
5. Press [▲▼] [▲▼] [2] (go to) [0] [0]	The step programmed at 00 will be displayed.

Checking the program

Each of the programs in the library is shown in check symbol form in the second column on the right-hand side of the page.

Press [▲▼] [C/CE] (step) repeatedly, and at each stage the check symbol will appear on the left of the display with the step number on the right. Ignore the four zeros in the display.

e.g. A.0000 03

check symbol — step number

After stepping through the program, press

[▲▼] [▲▼] [2] [0] [0] (go to) before execution.

Finally, press [C/CE] and the program is ready for use.

Correcting the program

If the check symbol for a particular step number is not as indicated in the last two columns of the program page:

1. Press [▲▼] [▲▼] [2] (go to) followed by the step number if the appropriate step number is not already displayed.
2. Press [▲▼] [RUN] (learn)
3. Enter the correct keystroke. The display will then show the next step in the program. If this is also incorrect, enter the correct keystroke. At each stage, the step about to be overwritten will be displayed.
4. When correction has been completed, press [C/CE]. Any step which has not been overwritten will not be affected.
5. Press [▲▼] [▲▼] [2] [0] [0] (go to)

Note

To restore normal use of the calculator after entering or checking the program, press [C/CE]

Running the program

Press the sequence of keys as shown in the program library in the execution sequence. Results displayed are printed in gold.

REACTANCES AND IMPEDANCES

Introduction

General note: conventions:

Voltage transfer ratios and current transfer ratios denoted by a_v and a_i are positive fractions $0 \leqslant a \leqslant 1$

Expressed in dB as gain, $A = 20 \log a$ is –ve

When expressed as an attenuation in dB, A is +ve and is given by $A = -20 \log a$

Power gain $= a_v a_i = a^2$, so $A = 10 \log (a^2) = 20 \log a$

Characteristic or design impedance $= R_o$

RESISTORS IN PARALLEL

(capacitors in series)
(inductors in parallel)
(conductors in series)

÷	G	00
+	E	01
rcl	5	02
=	–	03
sto	2	04
÷	G	05
=	–	06
stop	0	07
▼	A	08
goto	2	09
0	0	10
0	0	11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Pre-execution:

0 / ▲▼ / sto / C/CE / ▲▼ / ▲▼ / goto / 0 / 0 /

Execution:

R_1 / RUN / R_2 / RUN / $\frac{R_1 R_2}{R_1 + R_2}$ / R_3 / · · · / R_n /
RUN / $R_{parallel}$

Alternative execution:

To find resistor R_2 required to make parallel combination of R_1 and R_2 = R:

R / RUN / R_1 / ▲▼ / ▲▼ / +/– / RUN / R_2

(R_1 must be greater than R)

REACTANCE – FREQUENCY CONVERSIONS

$$X_C = \frac{1}{2\pi fC} = \frac{1}{\omega C} \qquad \text{(i)}$$

$$X_L = 2\pi fL = \omega L \qquad \text{(ii)}$$

$$C = \frac{1}{2\pi fX_C} = \frac{1}{\omega X_C} \qquad \text{(iii)}$$

$$L = \frac{X_L}{2\pi f} = \frac{X_L}{\omega} \qquad \text{(iv)}$$

$$f = \frac{1}{2\pi CX_C} \qquad \text{(v)}$$

$$f = \frac{X_L}{2\pi L} \qquad \text{(vi)}$$

Execution:

$$\text{f / RUN / } \left\{ \begin{array}{lll} & \div \text{ / RUN / } \omega & \\ \text{or} & \text{C / } \div \text{ / RUN / } X_C & \text{(i)} \\ \text{or} & \text{L / RUN / } X_L & \text{(ii)} \\ \text{or} & X_C \text{ / } \div \text{ / RUN / C} & \text{(iii)} \\ \text{or} & \div \text{ / } X_L \text{ / RUN / L} & \text{(iv)} \end{array} \right.$$

C / RUN / X_C / ÷ / RUN / f (v)

L / RUN / ÷ / X_L / RUN / f (vi)

X	·	00
#	3	01
6	6	02
·	A	03
2	2	04
8	8	05
3	3	06
1	1	07
8	8	08
5	5	09
3	3	10
÷	G	11
÷	G	12
stop	0	13
÷	G	14
=	—	15
stop	0	16
▼	A	17
goto	2	18
0	0	19
0	0	20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

MAGNITUDE AND PHASE OF IMPEDANCE

$$Z = R + jX = |Z|e^{j\phi}$$

$$|Z| = \sqrt{R^2 + X^2} \qquad \phi = \arctan\left(\frac{X}{R}\right)$$

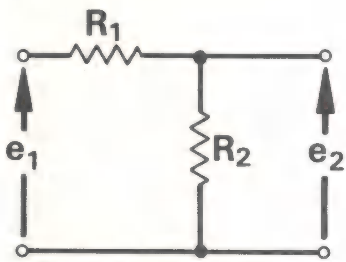
Execution:

X / RUN / R / RUN / |Z| / RUN / ϕ

For ϕ in degrees, insert / ▼ / R→D / after step 19.

sto	2	00
X	·	01
+	E	02
(	6	03
stop	0	04
÷	G	05
▼	A	06
MEx	5	07
÷	G	08
=	–	09
▼	A	10
arctan	9	11
▼	A	12
MEx	5	13
X	·	14
)	6	15
=	–	16
$\sqrt{x}$	1	17
stop	0	18
rcl	5	19
stop	0	20
▼	A	21
goto	2	22
0	0	23
0	0	24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

RESISTIVE VOLTAGE DIVIDER



To find R_1, R_2 given $R = R_1 + R_2$ and a or A

where $a = \frac{e_2}{e_1}$ $\qquad A = 20 \log \frac{e_2}{e_1}$

Execution:

R / RUN / a / RUN / R_2 / RUN / R_1

If A rather than a is given, see program on page 13.

–	F	00
(	6	01
X	·	02
stop	0	03
)	6	04
stop	0	05
=	–	06
stop	0	07
▼	A	08
goto	2	09
0	0	10
0	0	11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
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		24
		25
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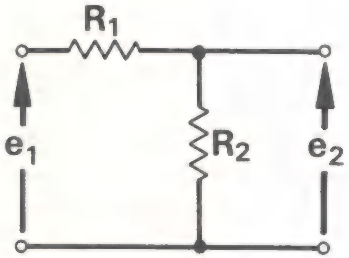
		33
		34
		35

RESISTIVE VOLTAGE DIVIDER



Given total resistance and attenuation, to find resistor values:

$R = R_1 + R_2$

$a = \frac{e_2}{e_1}$, $\quad A = 20 \log \frac{e_2}{e_1}$ dB

Execution:

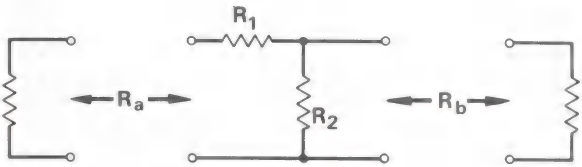
R / RUN / A / RUN / a / RUN / R_2 / RUN / R_1 / RUN / A / RUN / a / RUN / R_2 / RUN / R_1 / RUN / A / · · ·

If a is given, execute as below, or see shorter program on page 12.

R / RUN / ▲▼ / ▲▼ / goto / 13 / a / RUN / R_2 / RUN / R_1 / RUN / ▲▼ / ▲▼ / goto / 13 / a / RUN / R_2 / · · ·

sto	2	00
stop	0	01
÷	G	02
#	3	03
8	8	04
·	A	05
6	6	06
8	8	07
5	5	08
8	8	09
9	9	10
—	F	11
=	—	12
▼	A	13
e^x	4	14
stop	0	15
X	·	16
rcl	5	17
—	F	18
stop	0	19
rcl	5	20
—	F	21
=	—	22
stop	0	23
▼	A	24
goto	2	25
0	0	26
2	2	27
		28
		29
		30
		31
		32
		33
		34
		35

RESISTIVE L–PAD MATCHING IMPEDANCES



$$R_1 = \sqrt{R_a(R_a - R_b)} \qquad R_2 = \frac{R_a R_b}{R_1}$$

$$a_v = \frac{R_a - R_1}{R_a} \qquad A_v = 20 \log a_v$$

$$a_i = \frac{R_a}{R_a + R_1} \qquad A_i = 20 \log a_i$$

$$g = a_v a_i \qquad G = 10 \log a_v a_i$$

Pre-execution:

▲▼ / ▲▼ / goto / 0 / 0 / if previous run incomplete

sto	2	00
X	·	01
–	F	02
(	6	03
stop	0	04
X	·	05
rcl	5	06
)	6	07
sto	2	08
=	–	09
$\sqrt{x}$	1	10
stop	0	11
÷	G	12
X	·	13
rcl	5	14
X	·	15
stop	0	16
÷	G	17
X	·	18
rcl	5	19
+	E	20
sto	2	21
#	3	22
1	1	23
=	–	24
$\sqrt{x}$	1	25
–	F	26
(	6	27
▼	A	28
MEx	5	29
$\sqrt{x}$	1	30
)	6	31
÷	G	32
stop	0	33
=	–	34
stop	0	35

Execution:

R_a / RUN / R_b / RUN / R_1 / RUN / R_2 / RUN / $\sqrt{g}$ / and continue as required with one of the following sequences:

(i) To find a_v, A_v, A_i, G:

▲▼ / ▲▼ / MEx / RUN / a_v / ▲▼ / ln / X / 8·68589 / = / A_v

▲▼ / ▲▼ / MEx / ▲▼ / ln / X / 8·68589 / + / G

/ – / ▲▼ / rcl / = / A_i *or*

(ii) To find a_v:

/ ▲▼ / rcl / RUN / a_v *or*

(iii) To find a_i:

/ 1 / X / ▲▼ / rcl / RUN / a_i *or*

(iv) To find g:

/ 1 / X / RUN / g *or*

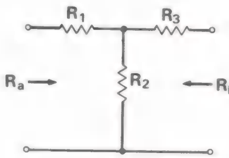
(v) To find a_v, g, a_1:

/ ▲▼ / ▲▼ / MEx / RUN / a_v

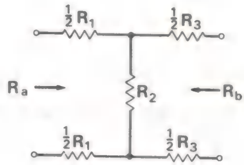
/ ▲▼ / ▲▼ / MEx / X / = / g

/ ÷ / ▲▼ / rcl / = / a_i

RESISTIVE ATTENUATOR SECTIONS, T–TYPE



Unbalanced T-network



Balanced H-network

$R_o = \sqrt{R_a R_b}\,, \qquad \rho = \frac{R_a}{R_o} = \frac{R_o}{R_b}$

Design attenuation = a (<1) = $\sqrt{a_v a_i}$

Power attenuation = A = –20 log a

Forward voltage transfer ratio $a_v = \frac{a}{\rho}$

Forward current transfer ratio $a_i = a\rho$

$$R_1 = \left[\frac{\rho(1 + a^2) - 2a}{1 - a^2}\right] R_o = (\rho k_1 - k_2)\, R_o$$

$$R_3 = \left[\frac{\frac{1}{\rho}(1 + a^2) - 2a}{1 - a^2}\right] R_o = \left(\frac{1}{\rho}k_1 - k_2\right) R_o$$

$$R_2 = \left[\frac{2a}{1 - a^2}\right] R_o = k_2 R_o$$

X	·	00
(	6	01
X	·	02
–	F	03
+	E	04
#	3	05
1	1	06
=	–	07
sto	2	08
÷	G	09
)	6	10
+	E	11
X	·	12
stop	0	13
–	F	14
stop	0	15
+	E	16
(	6	17
#	3	18
2	2	19
–	F	20
rcl	5	21
÷	G	22
rcl	5	23
X	·	24
stop	0	25
)	6	26
sto	2	27
=	–	28
stop	0	29
X	·	30
÷	G	31
X	·	32
rcl	5	33
–	F	34
stop	0	35

Pre-execution: use as required:

(i) given R_a and R_b, find and note ρ and R_o

R_a / ▲▼ / sto / ÷ / R_b / = / ▲▼ / $\sqrt{x}$ / ρ /
÷ / X / ▲▼ / rcl / = / R_o

(ii) given A, find and note a

A / – / ÷ / 8·68589 / = / ▲▼ / ▲▼ / e^x / a
▲▼ / ▲▼ / goto / 0 / 0 /

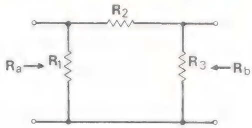
Execution:

a / RUN / k_2 / R_o / RUN / R_2 / RUN / k_1 / R_o /
X / ρ / RUN / R_1 / ρ / RUN / R_2 / = / R_3

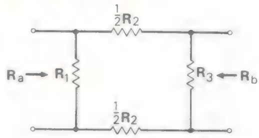
Special case, ρ = 1:

a / RUN / k_2 / R_o / RUN / R_2 / RUN / k_1 / R_o /
RUN / R_1 = R_3

RESISTIVE ATTENUATOR SECTIONS, π TYPE



Unbalanced π section



Balanced O section

$$R_o = \sqrt{R_a R_b} \qquad \rho = \frac{R_a}{R_o} = \frac{R_o}{R_b}$$

a_v = forward voltage transfer ratio $= \dfrac{a}{\rho}$

a_i = forward current transfer ratio $= a\rho$

a = design attenuation $= \sqrt{a_v a_i}$

A = power attenuation = $-20 \log a$ (in dB)

$$R_1 = \left[\frac{1 - a^2}{\frac{1}{\rho}(1 + a^2) - 2a}\right] R_o$$

$$R_3 = \left[\frac{1 - a^2}{\rho\,(1 + a^2) - 2a}\right] R_o$$

$$R_2 = \left[\frac{1 - a^2}{2a}\right] R_o$$

Pre-execution (as required):

(i) calculate and note ρ:

R_a / ▲▼ / sto / ÷ / R_b / = / ▲▼ / $\sqrt{x}$ / ρ

and continue to find R_o:

/ ÷ / X / ▲▼ / rcl / = / R_o

(ii) find and note a if given A:

/ A / – / ÷ / 8·68589 / = / ▲▼ / ▲▼ / e^x / a

set program:

▲▼ / ▲▼ / goto / 0 / 0 /

X	·	00
(	6	01
X	·	02
–	F	03
+	E	04
#	3	05
1	1	06
=	–	07
sto	2	08
÷	G	09
)	6	10
+	E	11
÷	G	12
X	·	13
stop	0	14
÷	G	15
stop	0	16
–	F	17
+	E	18
(	6	19
#	3	20
2	2	21
–	F	22
rcl	5	23
÷	G	24
rcl	5	25
÷	G	26
stop	0	27
)	6	28
sto	2	29
÷	G	30
=	–	31
stop	0	32
=	–	33
=	–	34
=	–	35

Execution:

a / RUN / R_o / RUN / R_2 / RUN / ρ / ÷ / R_o / RUN / R_1

Post-execution:

ρ / X / X / ▲▼ / rcl / – / ▲▼ / (/ R_2 / ÷ / ▲▼ /) / ÷ / = / R_3

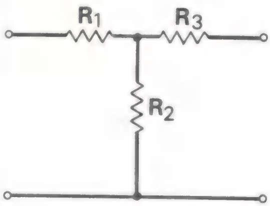
Special case : $\rho = 1$:

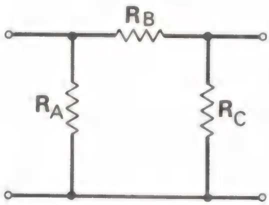
Execution:

a / RUN / R_o / RUN / R_2 / RUN / R_o / RUN / $R_1 = R_3$

RESISTOR NETWORKS

Π to T and T to Π transformations





$$R_o^2 = \frac{R_A R_B R_C}{R_A + R_B + R_C} = R_1 R_2 + R_2 R_3 + R_3 R_1$$

$$R_1 R_C = R_2 R_B = R_3 R_A = R_o^2$$

Execution:

(i) R_o known:

R_o / X / = / ▲▼ / sto / ▲▼ / ▲▼ / goto / 0 / 0 /

(ii) Π to T:

▲▼ / ▲▼ / goto / 0 / 9 / R_A / RUN / R_B / RUN / R_C / RUN / RUN /

(ii) T to Π:

▲▼ / ▲▼ / goto / 0 / 9 / R_1 / ÷ / RUN / R_2 / ÷ / RUN / R_3 / ÷ / RUN / ÷ / RUN /

Follow any of (i), (ii) or (iii) with either:

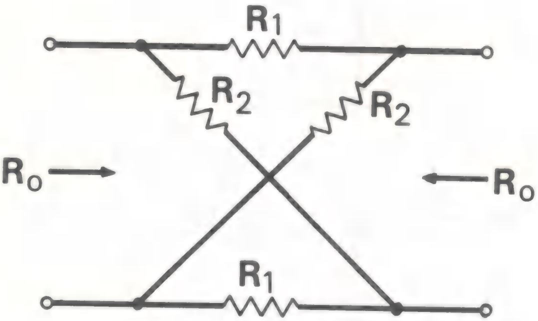
R_A / RUN / R_3 / R_B / RUN / R_2 / R_C / RUN / R_1

or:

R_1 / RUN / R_C / R_2 / RUN / R_B / R_3 / RUN / R_A

÷	G	00
X	·	01
rcl	5	02
=	–	03
stop	0	04
▼	A	05
goto	2	06
0	0	07
0	0	08
X	·	09
sto	2	10
(	6	11
stop	0	12
+	E	13
rcl	5	14
–	F	15
▼	A	16
MEx	5	17
)	6	18
X	·	19
(	6	20
stop	0	21
+	E	22
rcl	5	23
–	F	24
▼	A	25
MEx	5	26
)	6	27
÷	G	28
rcl	5	29
stop	0	30
=	–	31
sto	2	32
stop	0	33
=	–	34
=	–	35

LATTICE ATTENUATOR SECTIONS



(must be balanced, constant impedance)

$a_v = a_i = a$ $\qquad$ $A = -20 \log a$

Characteristic impedance = R_o

$R_1 = \dfrac{1-a}{1+a} R_o$ $\qquad$ $R_2 = \dfrac{1+a}{1-a} R_o$

Execution:

either

/ ▲▼ / ▲▼ / goto / 1 / 3 / a / RUN / R_o / RUN / R_2 / RUN / R_1

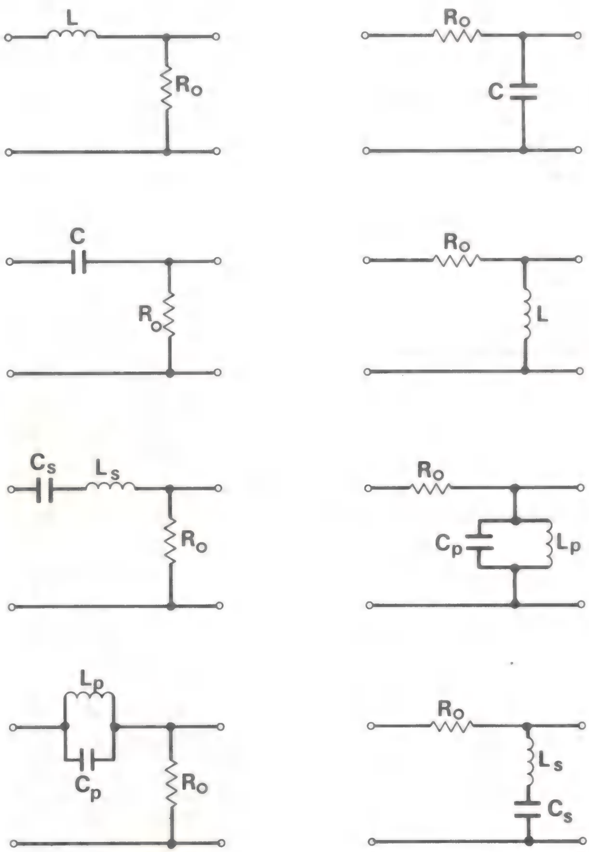
or

/ A / RUN / R_o / RUN / R_2 / RUN / R_1

–	F	00
÷	G	01
#	3	02
8	8	03
·	A	04
6	6	05
8	8	06
5	5	07
8	8	08
9	9	09
=	–	10
▼	A	11
e^x	4	12
+	E	13
#	3	14
1	1	15
÷	G	16
(	6	17
–	F	18
#	3	19
2	2	20
–	F	21
)	6	22
X	·	23
sto	2	24
stop	0	25
=	–	26
stop	0	27
÷	G	28
(	6	29
rcl	5	30
X	·	31
)	6	32
=	–	33
stop	0	34
=	–	35

FILTERS

Simple filters



Normalised to design impedance R_o,
ω_o = cut-off angular frequency (low-pass or high pass)

ω_o = centre frequency (band-pass or band stop)
ω_2 = upper cut-off frequency (band-pass or band stop)
ω_1 = lower cut-off frequency (band-pass or band stop)

$$\omega_o = \sqrt{\omega_1 \omega_2}$$

$$n = \frac{\omega_2 - \omega_1}{\omega_o}$$

Definitions:

x = normalised frequency parameter $= \dfrac{\omega}{\omega_o}$

v = deviation parameter = x (low pass) $= -\dfrac{1}{x}$ (high pass)

$$v = \frac{x - \dfrac{1}{x}}{n} \text{ (band pass)} \quad = \frac{n}{\dfrac{1}{x} - x} \text{ (band stop)}$$

Design:

Low-pass and high pass:

$$L = \frac{R_o}{\omega_o} \qquad C = \frac{1}{\omega_o R_o}$$

Use frequency-reactance conversion program (page 10)

Band-pass and band stop:

$$\omega_o \sqrt{L_p C_p} = \omega_o \sqrt{L_s C_s} = 1$$

$$L_s = \frac{L}{n}, \quad C_s = nC \qquad L_p = nL, \quad C_p = \frac{C}{n}$$

Use frequency-reactance conversion program (page 10)

FILTERS

Simple filters (contd.)

sto	2	00
–	F	01
(	6	02
rcl	5	03
÷	G	04
)	6	05
÷	G	06
stop	0	07
=	–	08
stop	0	09
sto	2	10
X	·	11
+	E	12
#	3	13
1	1	14
=	–	15
$\sqrt{x}$	1	16
ln	4	17
–	F	18
X	·	19
#	3	20
8	8	21
·	A	22
6	6	23
8	8	24
5	5	25
8	8	26
9	9	27
=	–	28
stop	0	29
rcl	5	30
▼	A	31
arctan	9	32
–	F	33
=	–	34
stop	0	35

Performance:

A = attenuation (dB) = $-8{\cdot}68589 \ln \sqrt{1 + v^2}$
ϕ = phase = –arctan v

Execution:

Band-pass:
x / RUN / n / RUN / v / RUN / A / RUN / ϕ

Band stop:
x / RUN / n / ÷ / – / RUN / v / RUN / A / RUN / ϕ

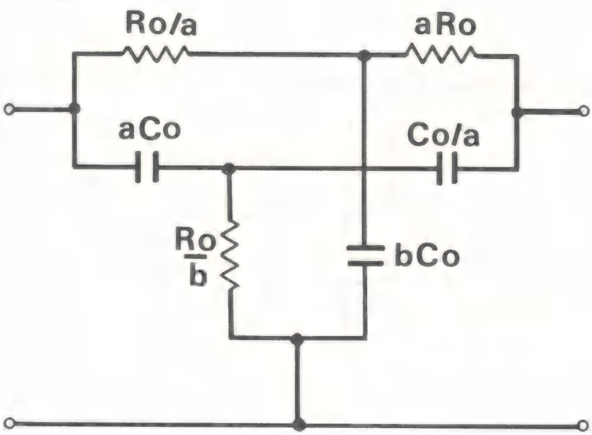
Low pass:
▲▼ / ▲▼ / goto / 1 / 0 / x / RUN / A / RUN / ϕ (v = x)

High pass:
▲▼ / ▲▼ / goto / 0 / 8 / x / ÷ / – / RUN / v / RUN / A / RUN / ϕ

To obtain x, pre-execution could be:
f / ÷ / f_o / = / or ω / ÷ / ω_o / = /

FILTERS

The twin-T network



Design:

ω_o = null frequency $\qquad x = \dfrac{\omega}{\omega_o}$

$\omega_o C_o R_o = 1$ (use reactance frequency program)

$b = a + \dfrac{1}{a}$ $\qquad v = -\dfrac{n}{x - \dfrac{1}{x}}$

$u = \dfrac{x - \dfrac{1}{x}}{b}$, where $n = \dfrac{2b}{a} = 2 + \dfrac{2}{a^2}$

$G_o = \dfrac{1}{R_o}$ $\qquad a = \sqrt{\dfrac{2}{n-2}}$

The twin-T network (contd.)

Performance:

The Y-matrix is, in terms or normalised variables:

$$Y = \frac{G_o}{1+jx}\begin{bmatrix} 2a + j\dfrac{x-\frac{1}{x}}{b} & -j\dfrac{x-\frac{1}{x}}{b} \\ -j\dfrac{x-\frac{1}{x}}{b} & \dfrac{2}{a} + j\dfrac{x-\frac{1}{x}}{b} \end{bmatrix} \qquad \Delta Y = \frac{2G_o^2}{jx}$$

$$= \frac{G_o}{1+jx}\begin{bmatrix} 2a + ju & -ju \\ -ju & \dfrac{2}{a} + ju \end{bmatrix}$$

with zero source impedance and load admittance (the usual conditions)

$$a_v = -\frac{y_{21}}{y_{22}} = \frac{ju}{\frac{2}{a} + ju} = \frac{1}{1+jv} = -\frac{2b}{a\left(x - \frac{1}{x}\right)}$$

Attenuation in dB = A = $-8{\cdot}68589 \ln \sqrt{1+v^2}$

Phase, $\phi = -\arctan v$

Use simple filters program with $n = \dfrac{2b}{a}$ (see page 24)

FILTERS

The twin-T network (contd.)

Design case:

Given:

Lower cut-off frequency = ω_1,
null frequency = ω_0.

Find:

a, hence component values, hence frequency response curve.

$$x_1 = \frac{\omega_1}{\omega_o} \qquad a = \sqrt{\frac{2}{\frac{1}{x_1} - x_1 - 2}} \qquad b = a + \frac{1}{a}$$

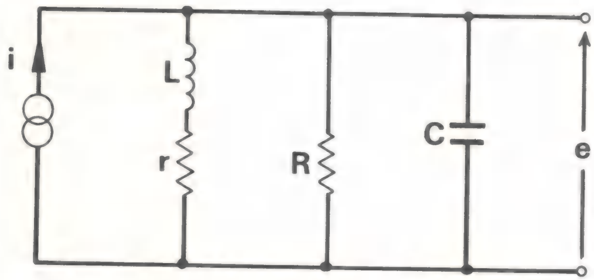
Execution:

x_1 / RUN / n / RUN / a / RUN / b

sto	2	00
÷	G	01
–	F	02
rcl	5	03
–	F	04
stop	0	05
#	3	06
2	2	07
÷	G	08
+	E	09
=	–	10
$\sqrt{x}$	1	11
stop	0	12
sto	2	13
÷	G	14
+	E	15
rcl	5	16
=	–	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

FILTERS

Single tuned circuit with losses



$$\omega_o = \frac{1}{\sqrt{LC}} \qquad R_o = \omega_o L = \frac{1}{\omega_o C} = \sqrt{\frac{L}{C}}$$

$$d_s = \frac{r}{\omega_o L} = \frac{r}{R_o} \qquad d_p = \frac{R_o}{R}$$

$$d = d_s + d_p \qquad Q = \frac{1}{d}$$

Normalised variables:

Normalised frequency $= x = \dfrac{\omega}{\omega_o}$

deviation $= v = Q\left(x - \dfrac{1}{x}\right)$

Normalised admittance:

$$y = YQR_o = \frac{1}{d}\left[d_p + \frac{d_s}{x^2 + d_s^2} + jx\left(1 - \frac{1}{x^2 + d_s^2}\right)\right]$$

Normalised impedance:

$$Z = \frac{1}{y} = \frac{e}{iQR_o} = \frac{Z}{QR_o} = d\left[d_p + \frac{d_s}{x^2 + d_s^2} + jx\left(1 - \frac{1}{x^2 + d_s^2}\right)\right]^{-1}$$

FILTERS

Single tuned circuit with losses (contd.)

For $Q \gg 1$, (or $Q > 10$), the frequency response is closely approximated by

$$\frac{e}{iR_o} = Q\,(1 + v^2)^{-1/2}$$

and can be found using the simple filters program.

For exact calculation, where $Q < 10$:

series resonant frequency = ω_o

$x_o = 1$

in-phase resonant frequency = ω_r

$x_r = \sqrt{1 - d_s^2}$

parallel resonant frequency = ω_p

$x_p = \left[(1 + 2d_s d_p + 2d_s^2)^{1/2} - d_s^2\right]^{1/2}$

impedance at ω_r = R_r = QR_o

Resonant frequencies

Execution:

d_s / RUN / x_r / d_p / RUN / x_p

sto	2	00
X	·	01
–	F	02
+	E	03
#	3	04
1	1	05
=	–	06
$\sqrt{x}$	1	07
stop	0	08
+	E	09
rcl	5	10
X	·	11
rcl	5	12
+	E	13
+	E	14
#	3	15
1	1	16
=	–	17
$\sqrt{x}$	1	18
–	F	19
(	6	20
rcl	5	21
X	·	22
)	6	23
=	–	24
$\sqrt{x}$	1	25
stop	0	26
▼	A	27
goto	2	28
0	0	29
0	0	30
		31
		32
		33
		34
		35

FILTERS

Single tuned circuit with losses (contd.)

Amplitude and phase response – Preliminary program

To find a and b:

$a = 2 + d_s^2 - d_p^2$

$b = 1 + 2d_p d_s + 2d_s^2$

Execution:

d_p / RUN / d_s / RUN / b / RUN / a

sto	2	00
X	·	01
—	F	02
+	E	03
(	6	04
stop	0	05
+	E	06
▼	A	07
MEx	5	08
X	·	09
rcl	5	10
+	E	11
+	E	12
#	3	13
1	1	14
=	—	15
stop	0	16
rcl	5	17
X	·	18
)	6	19
+	E	20
#	3	21
2	2	22
=	—	23
stop	0	24
▼	A	25
goto	2	26
0	0	27
0	0	28
		29
		30
		31
		32
		33
		34
		35

FILTERS

Single tuned circuit with losses (contd.)

Amplitude and phase response

$$|z| = d\left[u^2 - a + \frac{b}{u^2}\right]^{-1/2}$$

$$\phi = -\arctan \frac{x\,(u^2 - 1)}{u^2 d_p + d_s}$$

where $u^2 = x^2 + d_s^2$ $\qquad$ $d = d_s + d_p$

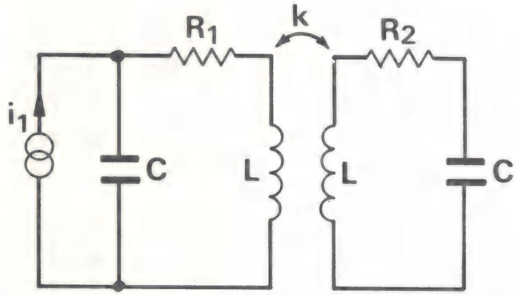
Execution:

x / RUN / d_s / RUN / b / RUN / a / RUN / d / $|z|$ / X / iQR_o / = / e / d_p / RUN / d_s / RUN / X / RUN / ▲▼ / ▲▼ / arctan / ϕ

X	·	00
+	E	01
(	6	02
stop	0	03
X	·	04
)	6	05
+	E	06
sto	2	07
(	6	08
÷	G	09
X	·	10
stop	0	11
)	6	12
–	F	13
stop	0	14
÷	G	15
=	–	16
$\sqrt{x}$	1	17
X	·	18
stop	0	19
X	·	20
rcl	5	21
+	E	22
stop	0	23
÷	G	24
X	·	25
(	6	26
#	3	27
1	1	28
–	F	29
rcl	5	30
)	6	31
X	·	32
stop	0	33
=	–	34
stop	0	35

TUNED COUPLED CIRCUITS

Response of secondary circuit



Case of two tuned circuits having equal inductances and capacitances but unequal Q-factors

Normalised response in secondary (relative to output at ω_o when s = 1)

$$y_2 = \frac{2s}{1 + s^2 + jvb - v^2} \quad \text{where}$$

$$v = \sqrt{Q_1 Q_2}\left(x - \frac{1}{x}\right) x = \frac{\omega}{\omega_o}$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$b = \left(\frac{Q_1}{Q_2} + \frac{Q_2}{Q_1}\right) \qquad Q_1 = \frac{\omega_o L}{R_1} \qquad Q_2 = \frac{\omega_o L}{R_2}$$

$$s = k\sqrt{Q_1 Q_2}$$

$$k = \text{coupling factor} = \frac{M}{\sqrt{L_1 L_2}} = \frac{M}{L}$$

$$a = \sqrt{b + 2}$$

X	·	00
+	E	01
#	3	02
1	1	03
–	F	04
(	6	05
stop	0	06
X	·	07
)	6	08
=	–	09
sto	2	10
stop	0	11
–	F	12
X	·	13
stop	0	14
÷	G	15
rcl	5	16
X	·	17
(	6	18
▼	A	19
arctan	9	20
stop	0	21
rcl	5	22
)	6	23
X	·	24
+	E	25
(	6	26
rcl	5	27
X	·	28
)	6	29
=	–	30
$\sqrt{x}$	1	31
÷	G	32
+	E	33
X	·	34
stop	0	35

Magnitude:

$$|y_2| = \frac{2s}{[(1+s^2-v^2)^2+b^2v^2]^{1/2}} = \frac{2s}{\left[(1+s^2)^2-2v^2\left(s^2-\frac{b}{2}\right)+v^4\right]^{1/2}}$$

Phase:

$$\phi = -\arctan \frac{v\sqrt{b+2}}{1+s^2-v^2}$$

Execution:

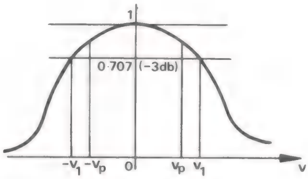
S / RUN / v / RUN / v / RUN / a / RUN / ϕ / RUN / s / = / $|y_2|$

Note: as $|v|$ increases, ϕ changes sign. Correct value of ϕ when this happens is obtained by subtracting π if v is positive, adding π if v is negative.

To obtain ϕ in degrees, use / ▲▼ / ▲▼ / R→D / before final / RUN /. Correct sign change by subtracting 180°.

TUNED COUPLED CIRCUITS

Design for linear phase response



Theory:

$$\phi = -\arctan \frac{v\sqrt{b+2}}{1+s^2-v^2}$$

$$\frac{d\phi}{dv} = -\frac{\sqrt{b+2}\,(1+s^2+v^2)}{(1+s^2)^2 - 2v^2\left(s^2-\frac{b}{2}\right) + v^4}$$

For maximally linear phase/frequency characteristic, the condition is:

$$s^2 = \frac{b-1}{3}$$

For maximum energy transfer the condition is s = 1 (critical coupling), hence to satisfy both conditions, b = 4 is optimum.

The frequency response is:

$$|y_2| = \frac{2s}{\left[\frac{(b+2)^2}{3} + v^2\left(\frac{b+2}{3}\right) + v^4\right]^{1/2}}$$

$$= \frac{2}{(4+2v^2+v^4)^{1/2}} \quad \text{for } b = 4$$

+	E	00
#	3	01
2	2	02
÷	G	03
#	3	04
3	3	05
—	F	06
sto	2	07
#	3	08
1	1	09
=	—	10
$\sqrt{x}$	1	11
stop	0	12
÷	G	13
(	6	14
×	·	15
—	F	16
+	E	17
rcl	5	18
)	6	19
×	·	20
(	6	21
#	3	22
3	3	23
×	·	24
rcl	5	25
=	—	26
$\sqrt{x}$	1	27
)	6	28
=	—	29
▼	A	30
arctan	9	31
▼	A	32
goto	2	33
1	1	34
2	2	35

$$\phi_2 = -\arctan \frac{v\sqrt{b+2}}{\frac{b+2}{3} - v^2} = -\arctan \frac{v\sqrt{6}}{2 - v^2}$$

Program computes s and ϕ_2 given b.

v_1 can be obtained by post-execution sequence.

Execution:

b / RUN / s / v / RUN / ϕ_2 (repeat for any other values of v)
/ v / RUN / ϕ_2 · · ·

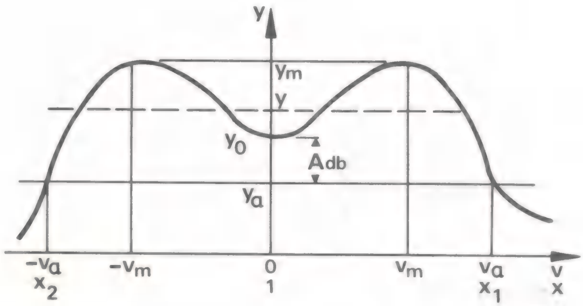
Bandwidth to 1% deviation from phase linearity: $v_p = {\cdot}49601\sqrt{1 + s^2}$

$\phi_2 = 1{\cdot}1394443$ for 1% deviation from phase linearity.

Attenuation at $v_p = -1{\cdot}1608$ dB relative to centre frequency.

TUNED COUPLED CIRCUITS –

Bandwidth to given attenuation



Let $\alpha = \dfrac{y_\alpha}{y_o}$, the attenuation at v_α relative to that at v = 0.

Then

$$v_\alpha{}^2 = \left(s^2 - \frac{b}{2}\right) \pm \sqrt{\left(s^2 - \frac{b}{2}\right)^2 + (1 + s^2)^2\left(\frac{1}{\alpha^2} - 1\right)}$$

The + sign gives values outside the peaks.

The – sign gives values inside the peaks, but only for $s^2 > \dfrac{b}{2}$ and $\alpha > 1$ (see dashed line).

If $y_\alpha > y_m$ or these conditions are not observed an error will be indicated.

$$v_m^2 = s^2 - \frac{b}{2}$$

X	·	00
÷	G	01
–	F	02
#	3	03
1	1	04
X	·	05
(	6	06
stop	0	07
X	·	08
+	E	09
sto	2	10
#	3	11
1	1	12
X	·	13
)	6	14
+	E	15
(	6	16
stop	0	17
–	F	18
÷	G	19
#	3	20
2	2	21
+	E	22
rcl	5	23
X	·	24
sto	2	25
)	6	26
=	–	27
$\sqrt{x}$	1	28
▼	A	29
MEx	5	30
stop	0	31
rcl	5	32
=	–	33
$\sqrt{x}$	1	34
stop	0	35

To find α from A dB:

A / – / ÷ / 8·68589 / = / ▲▼ / ▲▼ / e^x / α

Execution:

α / RUN / s / RUN / b / RUN / + / RUN / v_o outside peaks

α / RUN / s / RUN / b / RUN / – / RUN / v_u inside peaks

Error symbols:

If an error symbol occurs after / b / RUN / but before entering + or –, the value of α entered is too large (< ratio of peak to valley).

If an error symbol occurs after / d / – / RUN /, either

$s^2 \not> \frac{b}{2}$ or $\alpha < 1$.

Post execution:

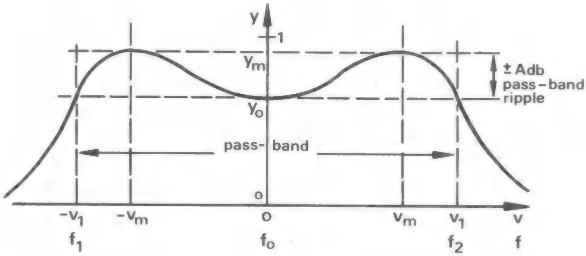
To find x from v:

v / ÷ / Q / X / ▲▼ / sto / – / 1 / = / ▲▼ / $\sqrt{x}$ / + / ▲▼ / rcl / = / x_1 / ÷ / = / x_2

(multiply x_1 or x_2 by f_o to obtain f_1 or f_2)

TUNED COUPLED CIRCUITS

Design for given bandwidth and pass-band ripple



Peak to valley ratio:

$$a = 10^{0\cdot1A} = e^{\frac{A}{4\cdot34294}}$$

$$a = \frac{y_m}{y_o} = \frac{1+s^2}{\left(1+s^2(b+2)-\frac{b^2}{4}\right)^{1/2}}$$

where $s = k\sqrt{Q_1 Q_2}$, $b = \frac{Q_1}{Q_2} + \frac{Q_2}{Q_1}$

∴ coupling for given peak to valley ratio:

$$s^2 = \frac{\frac{b}{2} + \sqrt{1-a^{-2}}}{1-\sqrt{1-a^{-2}}}$$

Location of peaks:

$$v_m = \sqrt{s^2 - \frac{b}{2}}$$

Location of pass-band edges:

$$v_1 = \sqrt{2s^2 - b} = \sqrt{2}\, v_m$$

×	·	00
÷	G	01
—	F	02
+	E	03
#	3	04
1	1	05
=	—	06
$\sqrt{x}$	1	07
sto	2	08
—	F	09
+	E	10
#	3	11
1	1	12
÷	G	13
(	6	14
stop	0	15
÷	G	16
#	3	17
2	2	18
+	E	19
▼	A	20
MEx	5	21
)	6	22
÷	G	23
—	F	24
▼	A	25
MEx	5	26
+	E	27
=	—	28
$\sqrt{x}$	1	29
stop	0	30
▼	A	31
MEx	5	32
$\sqrt{x}$	1	33
stop	0	34
=	—	35

Relation of Q to v, and band width:

$$Q = \sqrt{Q_1 Q_2} = \frac{v_1 f_o}{f_2 - f_1}$$

$$x = \frac{\omega}{\omega_o} = \frac{f}{f_o}$$

$$v = Q\left(x - \frac{1}{x}\right)$$

f_2 = upper limit of pass-band
f_1 = lower limit of pass-band
f_o = centre frequency = $\sqrt{f_1 f_2}$

To find a from A:

A / ÷ / 4·34294 / = / ▲▼ / ▲▼ / e^x / a

Execution:

Either:

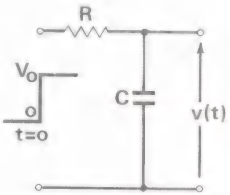
a / RUN / b / RUN / v_1 / X / f_o / ÷ / ▲▼ / (/ f_2 / – / f_1 / ▲▼ /) / = /
Q / RUN / s / ÷ / ▲▼ / rcl / = / k

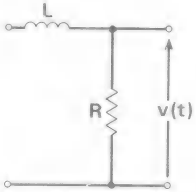
Or:

a / RUN / b / RUN / v_1 / RUN / s

LINEAR CIRCUIT THEORY

Simple L–R or C–R circuit





$\tau = CR$ or $\tau = \dfrac{L}{R}$

Charge: $V_c(t) = V_o(1 - e^{-\frac{t}{\tau}})$

Discharge: $V_d(t) = V_o e^{-\frac{t}{\tau}}$

Pre-execution:

R / X / C / = / ▲▼ / sto / *or*
L / ÷ / R / = / ▲▼ / sto / *or*
τ / ▲▼ / sto / ▲▼ / ▲▼ / goto / 0 / 0 /

Execution:

t / RUN / V_o / RUN / $V_d(t)$

÷	G	00
rcl	5	01
—	F	02
=	—	03
▼	A	04
e^x	4	05
X	·	06
stop	0	07
=	—	08
stop	0	09
▼	A	10
goto	2	11
0	0	12
0	0	13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

LINEAR CIRCUIT THEORY

Simple L–R or C–R circuit (contd.)

÷	G	00
rcl	5	01
—	F	02
=	—	03
▼	A	04
e^x	4	05
—	F	06
+	E	07
#	3	08
1	1	09
X	·	10
stop	0	11
=	—	12
stop	0	13
▼	A	14
goto	2	15
0	0	16
0	0	17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Pre-execution:

R / X / C / = / ▲▼ / sto / *or*
L / ÷ / R / = / ▲▼ / sto / *or*
τ / ▲▼ / sto / ▲▼ / ▲▼ / goto / 0 / 0 /

Execution:

t / RUN / V_o / RUN / V_c (t)

LINEAR CIRCUIT THEORY

Simple L–R or C–R circuit (contd.)

÷	G	00
rcl	5	01
–	F	02
=	–	03
▼	A	04
e^x	4	05
X	·	06
stop	0	07
–	F	08
stop	0	09
–	F	10
=	–	11
stop	0	12
▼	A	13
goto	2	14
0	0	15
0	0	16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Pre-execution:

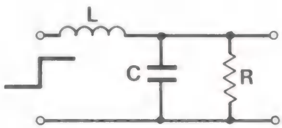
R / X / C / = / ▲▼ / sto / *or*
L / ÷ / R / = / ▲▼ / sto / *or*
τ / ▲▼ / sto / ▲▼ / ▲▼ / goto / 0 / 0 /

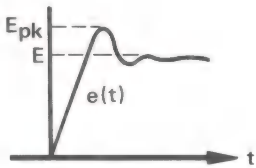
Execution:

t / RUN / V_o / RUN / $V_a(t)$ / V_o / RUN / $V_c(t)$

LINEAR CIRCUIT THEORY

Damping factor from transient response





$$\text{overshoot } (y) = \left(\frac{E_{pk}}{E} - 1\right) \qquad 0 \leqslant y \leqslant 1$$

$$K = \frac{X}{\sqrt{\pi^2 + X^2}} \text{ where } X = -\ln\left(\frac{E_{pk}}{E} - 1\right)$$

Note: This formula applies to ideal 2nd-order systems of all kinds.

Pre-execution:

To enter first set of values

▲▼ / ▲▼ / goto / 0 / 0 /

Execution:

E_{pk} / RUN / E / RUN / y / RUN / k

E'_{pk} / RUN / y' / RUN / k'

(continue for other values of E_{pk} at same E)

–	F	00
stop	0	01
sto	2	02
÷	G	03
rcl	5	04
=	–	05
stop	0	06
ln	4	07
–	F	08
÷	G	09
#	3	10
3	3	11
·	A	12
1	1	13
4	4	14
1	1	15
5	5	16
9	9	17
3	3	18
÷	G	19
(	6	20
X	·	21
+	E	22
#	3	23
1	1	24
=	–	25
$\sqrt{x}$	1	26
)	6	27
=	–	28
stop	0	29
–	F	30
rcl	5	31
▼	A	32
goto	2	33
0	0	34
3	3	35

LINEAR CIRCUIT THEORY

Time taken to reach given voltage

$$t_d = -\tau \ln \frac{v_d(t)}{V_o} \quad , \quad t_c = -\tau \ln \left(1 - \frac{v_c(t)}{V_o}\right)$$

Pre-execution:

$-\tau$ / ▲▼ / sto / *or*

L / + / R / = / ▲▼ / sto / *or*

τ / ▲▼ / sto / ▲▼ / ▲▼ / goto / 0 / 0 /

Execution:

v(t) / RUN / V_o / RUN / t_d / RUN / t_c

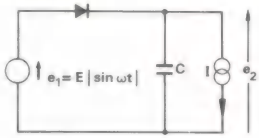
Special case:

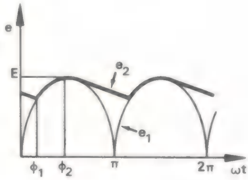
Rise-time –

Compute for $v(t) = 0{\cdot}1V_o \quad t_r = t_d - t_c = 2{\cdot}19714\tau$

÷	G	00
stop	0	01
−	F	02
(	6	03
ln	4	04
×	·	05
rcl	5	06
=	—	07
stop	0	08
#	3	09
1	1	10
=	—	11
)	6	12
−	F	13
=	—	14
ln	4	15
×	·	16
rcl	5	17
=	—	18
stop	0	19
▼	A	20
goto	2	21
0	0	22
0	0	23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

FULL–WAVE RECTIFIER WITH CAPACITOR SMOOTHING





sto	2	00
▼	A	01
arccos	8	02
X	·	03
▼	A	04
MEx	5	05
–	F	06
+	E	07
(	6	08
rcl	5	09
sin	7	10
)	6	11
=	–	12
stop	0	13
sto	2	14
cos	8	15
X	·	16
rcl	5	17
–	F	18
(	6	19
rcl	5	20
sin	7	21
)	6	22
+	E	23
stop	0	24
÷	G	25
(	6	26
rcl	5	27
cos	8	28
+	E	29
stop	0	30
▼	A	31
goto	2	32
1	1	33
1	1	34
		35

The diode conducts from ϕ_1 to ϕ_2 in each input cycle where

$$\cos \phi_2 = -\frac{I}{\omega CE} = -x$$

$$\sin \phi_1 + x\,\phi_1 = \sin(\arccos x) - x \arccos x = k$$

This program finds ϕ_2 and then calculates ϕ_1 using the Newton-Raphson iterative formula

$$\phi_1' = \frac{\phi_1 \cos \phi_1 - \sin \phi_1 + k}{\cos \phi_1 + x}$$

Pre-execution:

▲▼ / ▲▼ / goto / 0 / 0 /

Execution:

x / RUN / k

3·14159 / – / ▲▼ / rcl / = / ϕ_2

/ ▲▼ / rcl / $\pi - \phi_2$ (used as starting value ϕ_1)

ϕ_1 / RUN / k / RUN / x / RUN / ϕ_1'

repeat until convergence obtained.
(ϕ_1 is also in memory)

Given ϕ_1 and ϕ_2 all the useful circuit parameters can be calculated. (see over)

RECTIFIER WITH CAPACITIVE SMOOTHING

Ripple voltage:

$v_{r\ pk\text{-}pk} = E\ (1 - \sin \phi_1)$

Post execution:

▲▼ / sin / – / + / 1 / X / E / = / $v_{r\ pk\text{-}pk}$

Peak rectifier current:

$i_{d\ pk} = I + \omega CE \cos \phi_1 = I \left(1 + \dfrac{\cos \phi_1}{x} \right)$

Post execution:

▲▼ / rcl / ▲▼ / cos / ÷ / x / + / 1 / X / I / = / $i_{d\ pk}$

RECTIFIER WITH CAPACITIVE SMOOTHING

Calculate ϕ_1 and ϕ_2 using program given (page 45).

Mean rectified voltage:

$$\overline{e_2} = \frac{2}{\pi} E \sin a \, (\cos b' + b' \sin b')$$

$$\text{when } a = \frac{\phi_1 + \phi_2}{2}, \quad b' = \frac{\phi_1 + \pi - \phi_2}{2}$$

$$b = \frac{\phi_1 - \phi_2}{2}$$

Pre-execution:

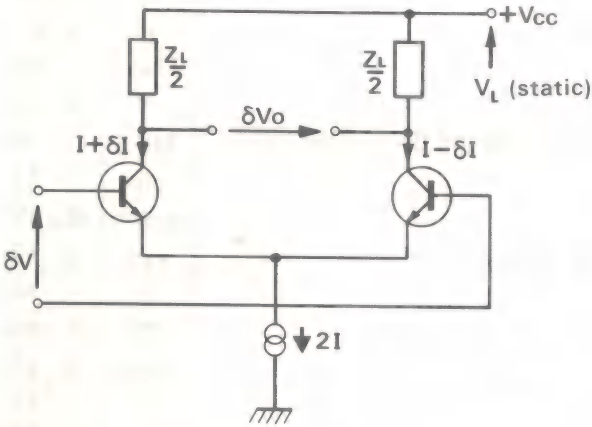
ϕ_1 / + / ϕ_2 / ▲▼ / sto / ÷ / 2 / – / a / ▲▼ / ▲▼ / MEx / + / b

Execution:

/ RUN / E / = / $\overline{e_2}$

(	6	00
#	3	01
1	1	02
·	A	03
5	5	04
7	7	05
0	0	06
8	8	07
=	–	08
▼	A	09
MEx	5	10
sin	7	11
÷	G	12
rcl	5	13
=	–	14
▼	A	15
MEx	5	16
)	6	17
=	–	18
▼	A	19
MEx	5	20
X	·	21
(	6	22
rcl	5	23
sin	7	24
X	·	25
rcl	5	26
=	–	27
▼	A	28
MEx	5	29
cos	8	30
+	E	31
rcl	5	32
)	6	33
X	·	34
stop	0	35

TRANSFER FUNCTION OF LONG–TAILED PAIR



$$\delta V = \frac{KT}{q} \ln \left(\frac{1 + \frac{\delta I}{I}}{1 - \frac{\delta I}{I}} \right)$$

$$\frac{\delta I}{I} = \frac{\exp\left(\frac{q\delta V}{kT}\right) - 1}{\exp\left(\frac{q\delta V}{kT}\right) + 1}$$

$\delta V_o = Z_L \delta I$

q = electronic charge = $1{\cdot}602192 \times 10^{-19}$ C

k = Boltzmann's constant = $1{\cdot}380622 \times 10^{-23}\ \mathrm{JK^{-1}}$

T = absolute temperature (°C + 273·15)

$V_L = \frac{IR_L}{2}$ (if load is resistive)

X	·	00
#	3	01
8	8	02
·	A	03
6	6	04
1	1	05
7	7	06
1	1	07
·	A	08
·	A	09
5	5	10
=	–	11
sto	2	12
stop	0	13
÷	G	14
rcl	5	15
=	–	16
▼	A	17
e^x	4	18
–	F	19
#	3	20
1	1	21
÷	G	22
(	6	23
+	E	24
#	3	25
2	2	26
=	–	27
)	6	28
X	·	29
stop	0	30
=	–	31
▼	A	32
goto	2	33
1	1	34
3	3	35

(set temperature:)

Pre-execution:

▲▼ / ▲▼ / goto / 0 / 0 / T / RUN

Execution:

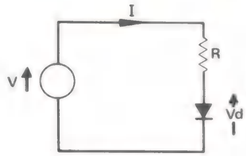
$$\delta V \,/\, \text{RUN} \,/\, \frac{\delta I}{I} \quad \left\{ \begin{array}{l} I \,/\, \text{RUN} \,/\, \delta I \\ I \,/\, X \,/\, Z_L \,/\, \text{RUN} \,/\, \delta V_D \\ V_L \,/\, + \,/\, \text{RUN} \,/\, \delta V_a \end{array} \right\}$$

Repeat for all required values of δV

e.g. for sine wave, $\delta V = V \sin \omega t$,

/ ω / X / t / = /▲▼/ sin / X / V / RUN / I / RUN / δI etc.

OPERATING POINT OF DIODE–RESISTOR COMBINATION



$$V = IR + \frac{nkT}{q} \ell n \left(1 + \frac{I}{I_s}\right)$$

Newton-Raphson method gives the iteration formula for I

$$I' = \frac{V + \frac{nkT}{q}\left(\frac{I}{I + I_s}\right) - \frac{nkT}{q} \ell n \left(1 + \frac{I}{I_s}\right)}{R + \frac{nkT}{q}\left(\frac{I}{I + I_s}\right)}$$

For forward-biased diodes, $I \gg I_s$, so this simplifies to

$$I' \doteq \frac{V + \frac{nkT}{q}\left(1 - \ell n \frac{I}{I_s}\right)}{R + \frac{nkT}{qI}}$$

If I is mA and V_o = diode voltage at I_o = 1mA,

$$I' \doteq \frac{V - V_o + \frac{nkT}{q}\left(1 - \ell n \frac{I}{I_o}\right)}{R + \frac{nkT}{qI}}$$

where $V_o = \frac{nkT}{q} \ell n \frac{I_o}{I_s}$

÷	G	00
X	·	01
(	6	02
ln	4	03
sto	2	04
#	3	05
·	A	06
0	0	07
8	8	08
6	6	09
1	1	10
7	7	11
1	1	12
X	·	13
stop	0	14
X	·	15
▼	A	16
MEx	5	17
+	E	18
rcl	5	19
+	E	20
stop	0	21
=	–	22
▼	A	23
MEx	5	24
)	6	25
+	E	26
stop	0	27
÷	G	28
rcl	5	29
÷	G	30
=	–	31
=	–	32
=	–	33
=	–	34
stop	0	35

Consistent units are:

V in mV, R in Ω, I in mA

n = 1 for germanium diodes or for transistor junctions

n = 1·5 for silicon p-n diodes

Find $\dfrac{nkT}{q}$ to use in program (in mV)

or use T each time in program execution if desired.

Execution:

(with T) ▲▼ / ▲▼ / goto / 0 / 0 /

$$\text{I / RUN /}\left\{\begin{array}{l}\text{T / RUN /}\\ \text{T / X / n / RUN /}\end{array}\right\}\left\{\begin{array}{l}\text{V / – / }V_o\\ V - V_o\end{array}\right\}\text{/ RUN / R / RUN / }I'$$

$$\text{/RUN /}\left\{\begin{array}{l}\text{T}\\ \text{T / X / n}\end{array}\right\}\text{/ RUN /}\left\{\begin{array}{l}\text{V / – / }V_o\\ V - V_o\end{array}\right\}\text{/ RUN / R / RUN / }I''$$

(repeat until values converge)

(without T) – enter a constant as indicated (to lesser accuracy as required) at steps 06 to 14

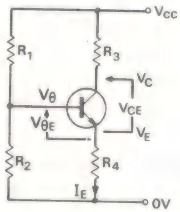
$$\text{I / RUN /}\left\{\begin{array}{l}V - V_o\\ \text{V / – / }V_o\end{array}\right\}\text{/ RUN / R / RUN / }I'$$

$$\text{/ RUN /}\left\{\begin{array}{l}V - V_o\\ \text{V / – / }V_o\end{array}\right\}\text{/ RUN / R / RUN / }I''$$

($\dfrac{nkT}{q}$ may be found from: / n / X / T / X / 1·086171 / = / $\dfrac{nkT}{q}$ mV;

at 25°C $\dfrac{kT}{q} \triangleq 25{\cdot}6789$ mV)

OPERATING POINT OF TRANSISTOR IN BASE–POTENTIAL DIVIDER AND EMITTER RESISTOR BIAS



Preliminary equations:

$$V = \frac{V_{CC} R_2}{R_1 + R_2}$$

$$R = R_4 + \frac{R_1 R_2}{(R_1 + R_2)(h_{FE} + 1)}$$

I_E is given by the diode-resistor program with $V_o = V_{BE}$ of transistor at 1 mA, R and V as given above, and n = 1.

Circuit equations:

$$V_E = I_E R_4 \qquad I_C = I_E \frac{h_{FE}}{1 + h_{FE}}$$

$$V_{BE} = \frac{k}{q} \ln I_E\,(\text{mA}) + V_o$$

$$V_B = V_E + V_{BE}$$

$$V_C = V_{CC} - I_E R_3 \frac{h_{FE}}{1 + h_{FE}}$$

$$V_{CE} = V_C - V_E$$

Prelim. program

+	E	00
stop	0	01
sto	2	02
÷	G	03
(	6	04
–	F	05
rcl	5	06
)	6	07
÷	G	08
X	·	09
▼	A	10
MEx	5	11
÷	G	12
stop	0	13
+	E	14
stop	0	15
=	–	16
stop	0	17
X	·	18
rcl	5	19
–	F	20
stop	0	21
=	–	22
stop	0	23
▼	A	24
goto	2	25
0	0	26
0	0	27
		28
		29
		30
		31
		32
		33
		34
		35

Final program

sto	2	00
ln	4	01
X	·	02
#	3	03
·	A	04
0	0	05
8	8	06
6	6	07
1	1	08
7	7	09
1	1	10
X	·	11
stop	0	12
+	E	13
stop	0	14
+	E	15
(	6	16
stop	0	17
X	·	18
rcl	5	19
)	6	20
stop	0	21
=	—	22
stop	0	23
÷	G	24
(	6	25
+	E	26
#	3	27
1	1	28
=	—	29
)	6	30
—	F	31
X	·	32
rcl	5	33
X	·	34
stop	0	35

1. Enter preliminary program . . .

Execution:

R_2 / RUN / R_1 / RUN / h_{FE} + 1 / RUN / R_4 / RUN / R

V_{CC} / RUN / V / V_o / RUN / $V - V_o$

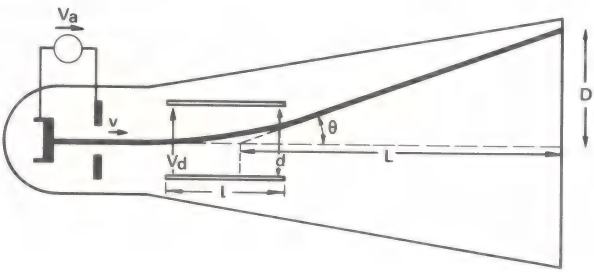
2. Next enter diode and resistor program (see page 50) and execute to find I_E

3. Finally enter program in box and run:

Execution:

I / RUN / T / RUN / V_o / RUN / V_{BE} / R_4 / RUN / V_E / RUN / V_B / h_{FE} / RUN / $-I_C$ / R_3 / + / V_{CC} / – / V_C / V_E / = / V_{CE}

ELECTRON DYNAMICS



(S.I. Units)

To find electrostatic deflection, velocity, sensitivity, deflection and angle of deflection in cathode ray tube.
(non-relativistic)

$$v = \sqrt{\frac{2eV_a}{m}}$$

$$S = \frac{lL}{2dV_a}$$

$$D = \frac{lLV_d}{2dV_a} = SV_d$$

$$\theta = \arctan\frac{D}{L} = \arctan\frac{lV_d}{2dV_a}$$

e = electron charge = $1{\cdot}6022 \times 10^{-19}$ C

m = electron mass = $9{\cdot}1096 \times 10^{-31}$ kg

Execution:

V_a / RUN / v / d / RUN / l / RUN / L / RUN / S / V_d / RUN / D / RUN / θ

sto	2	00
$\sqrt{x}$	1	01
X	·	02
#	3	03
5	5	04
·	A	05
9	9	06
3	3	07
0	0	08
9	9	09
·	A	10
5	5	11
=	—	12
stop	0	13
+	E	14
÷	G	15
X	·	16
stop	0	17
÷	G	18
rcl	5	19
X	·	20
stop	0	21
sto	2	22
X	·	23
stop	0	24
÷	G	25
stop	0	26
rcl	5	27
=	—	28
▼	A	29
arctan	9	30
stop	0	31
▼	A	32
goto	2	33
0	0	34
0	0	35

DEFLECTION OF RELATIVISTIC ELECTRONS

Small transverse field ■ in cathode ray tube

$$\frac{D}{L} = \tan\theta \triangleq \frac{eV_d}{mc^2}\,\frac{l}{d} \times \left[\left(1+\frac{eV_a}{mc^2}\right) - \left(1+\frac{eV_a}{mc^2}\right)^{-1}\right]^{-1}$$

Execution:

for D or θ only –

V_a / RUN / V_d / RUN / d / RUN / l / RUN /

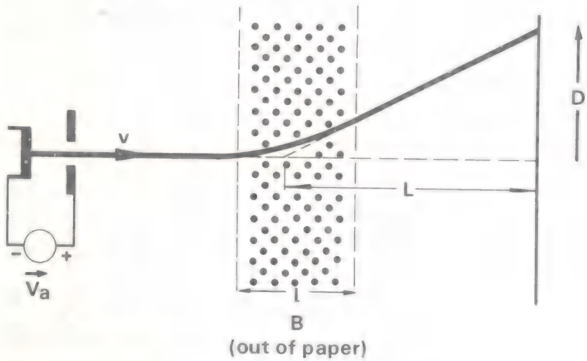
$\tan\theta \begin{cases} / \times / L / = / D \\ / \text{RUN} / \theta \end{cases}$

or, for S, D and θ

V_a / RUN / L / RUN / d / RUN / l / RUN / S /
× / V_d / ÷ / D / L / = / $\tan\theta$ / RUN / θ

×	·	00
(	6	01
#	3	02
1	1	03
·	A	04
9	9	05
5	5	06
6	6	07
9	9	08
·	A	09
·	A	10
6	6	11
=	–	12
sto	2	13
)	6	14
+	E	15
#	3	16
1	1	17
–	F	18
(	6	19
÷	G	20
)	6	21
÷	G	22
×	·	23
rcl	5	24
×	·	25
stop	0	26
÷	G	27
stop	0	28
×	·	29
stop	0	30
=	–	31
stop	0	32
▼	A	33
arctan	9	34
stop	0	35

MAGNETIC DEFLECTION IN CATHODE–RAY TUBE (non-relativistic)



$$\theta = \arcsin \frac{leB}{mv} = \arcsin \frac{lB}{\sqrt{V_a}} \sqrt{\frac{e}{2m}}$$

$$D = L \tan \theta$$

$$S = \frac{D}{B} \bumpeq \frac{lL}{\sqrt{V_a}} \sqrt{\frac{e}{2m}}$$ (magnetic deflection sensitivity for small θ)

Execution:

V / RUN / l / RUN / B / RUN / θ / RUN / L / RUN / S / RUN / D

Notes:

1. In practical wide angle tubes the field will not be uniform.
2. If $\theta > \frac{\pi}{2}$ is computed, a value of 0 with no error symbol will be shown. This means the electron is reversed in direction by the field.

$\sqrt{x}$	1	00
÷	G	01
×	·	02
#	3	03
2	2	04
9	9	05
6	6	06
5	5	07
4	4	08
6	6	09
×	·	10
stop	0	11
×	·	12
sto	2	13
stop	0	14
=	–	15
▼	A	16
arcsin	7	17
stop	0	18
tan	9	19
×	·	20
(	6	21
stop	0	22
×	·	23
▼	A	24
MEx	5	25
=	–	26
stop	0	27
rcl	5	28
)	6	29
=	–	30
stop	0	31
▼	A	32
goto	2	33
0	0	34
0	0	35

VELOCITY OF ACCELERATED ION (non-relativistic)

M = mass of ion
ne = charge on ion
V = accelerating potential (volts)

$$v = \sqrt{\frac{2neV}{M}}$$

Execution:

V / RUN / n / RUN / M / RUN / v

X	·	00
#	3	01
3	3	02
·	A	03
2	2	04
0	0	05
4	4	06
4	4	07
·	A	08
·	A	09
1	1	10
9	9	11
X	·	12
stop	0	13
÷	G	14
stop	0	15
=	—	16
$\sqrt{x}$	1	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

MASS AND VELOCITY OF ACCELERATED ELECTRON OR ION (relativistic)

V = accelerating potential (volts)

$$m_r = m\left(1 + \frac{eV}{mc^2}\right)$$

$$v_r = c\sqrt{1 - \left(1 + \frac{eV}{mc^2}\right)^{-2}}$$

For electron

$e = 1{\cdot}6022 \times 10^{-19}\,\mathrm{C}$

$m = 9{\cdot}1096 \times 10^{-31}\,\mathrm{kg}$

$c = 2{\cdot}9979 \times 10^{8}\,\mathrm{ms}^{-1}$

$$\frac{e}{mc^2} = 1{\cdot}9569 \times 10^{-6}\,\mathrm{V}^{-1}$$

Execution:

V / RUN / v_r / ▲▼ / rcl / X / 9·1096 / • / • / 31 / = / m_r

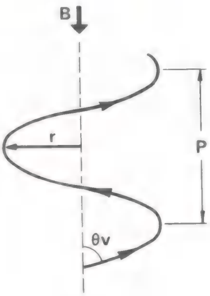
For ion of mass M and charge ne:

n / X / V / X / m / ÷ / M / RUN / v_r / ▲▼ / rcl / X / M / = / M_r

X	·	00
#	3	01
1	1	02
·	A	03
9	9	04
5	5	05
6	6	06
9	9	07
·	A	08
·	A	09
6	6	10
+	E	11
#	3	12
1	1	13
=	–	14
sto	2	15
÷	G	16
X	·	17
–	F	18
+	E	19
#	3	20
1	1	21
=	–	22
$\sqrt{x}$	1	23
X	·	24
#	3	25
2	2	26
·	A	27
9	9	28
9	9	29
7	7	30
9	9	31
·	A	32
8	8	33
=	–	34
stop	0	35

ELECTRON MOTION IN TRANSVERSE MAGNETIC FIELD

Radius and period of orbit, pitch of helical path.



Period $T = \dfrac{2\pi m}{eB}$ radius of circular path $r_c = \dfrac{vT}{2\pi}$

Radius of path r =

$$\frac{mv}{eB}\sin\theta = \frac{\sqrt{2m}}{e}\frac{\sqrt{V}}{B}\sin\theta = \frac{vT}{2\pi}\sin\theta$$

Pitch of path P =

$$\frac{2\pi mv}{eB}\cos\theta = 2\pi\frac{\sqrt{2m}}{e}\frac{\sqrt{V}}{B}\cos\theta = vT\cos\theta$$

θ = angle of injection (relative to B)

$$\left(\frac{2\pi m}{e} \doteqdot 3{\cdot}5724 \times 10^{-11}\right)$$

Pre-execution (if desired):

V / ▲▼ / $\sqrt{x}$ / X / 5·9309·5 / = / v

Execution:

v / RUN / B / = / T / RUN / r_c / θ / RUN / r /
θ / RUN / = / P

X	·	00
(	6	01
#	3	02
3	3	03
·	A	04
5	5	05
7	7	06
2	2	07
4	4	08
·	A	09
·	A	10
1	1	11
1	1	12
÷	G	13
stop	0	14
)	6	15
÷	G	16
sto	2	17
#	3	18
6	6	19
·	A	20
2	2	21
8	8	22
3	3	23
2	2	24
X	·	25
(	6	26
stop	0	27
sin	7	28
)	6	29
=	—	30
stop	0	31
cos	8	32
X	·	33
rcl	5	34
stop	0	35

CAPACITANCE OF SPHERE, CONCENTRIC SPHERES, CONCENTRIC CYLINDERS

(i) Sphere of radius a:

$$C = 4\pi\epsilon_o \epsilon_r a$$

(ii) Concentric spheres of radii a and b (b > a)

$$C = 4\pi\epsilon_o \epsilon_r \frac{ab}{b-a}$$

(iii) Concentric cylinders of radii a and b (b > a), and length L:

$$C = \frac{4\pi\,\epsilon_o \epsilon_r L}{2\ln\left(\frac{b}{a}\right)}$$

Pre-execution and execution:

(i) Sphere:

▲▼ / ▲▼ / goto / 1 / 9 / a / RUN / ϵ_r / RUN / C

(ii) Concentric spheres:

▲▼ / ▲▼ / goto / 1 / 2 / a / RUN / b / RUN / ϵ_r / RUN / C

(iii) Concentric cylinders:

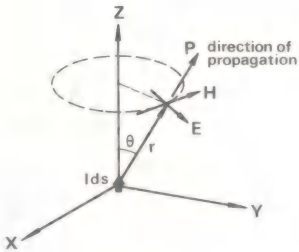
▲▼ / ▲▼ / goto / 0 / 0 / b / RUN / a / RUN / L / RUN / ϵ_r / RUN / C

($4\pi\epsilon_o = 1{\cdot}11265 \times 10^{-10}\,F\,m^{-1}$)

(S.I. units)

÷	G	00
stop	0	01
=	–	02
ln	4	03
+	E	04
÷	G	05
X	·	06
stop	0	07
▼	A	08
goto	2	09
1	1	10
9	9	11
÷	G	12
–	F	13
(	6	14
stop	0	15
÷	G	16
)	6	17
÷	G	18
X	·	19
#	3	20
1	1	21
·	A	22
1	1	23
1	1	24
2	2	25
6	6	26
5	5	27
·	A	28
·	A	29
1	1	30
0	0	31
X	·	32
stop	0	33
=	–	34
stop	0	35

FIELD STRENGTH AND POYNTING VECTOR DUE TO ELECTRIC DIPOLE



$$H = \frac{Ids}{2\lambda r} \sin\theta \sin\left(\omega t - \frac{2\pi r}{\lambda}\right)$$

$E = Z_i H$ where $Z_i = \sqrt{\dfrac{\mu_o}{\epsilon_o}} = \mu_o c \triangleq 376{\cdot}73\Omega$

$P = EH$ (power flow per unit area)

$$P_{av} = \frac{E_{pk} H_{pk}}{2}$$

$\lambda = \dfrac{c}{f}$ where $c = 2{\cdot}9979 \times 10^8\ ms^{-1}$

Execution:

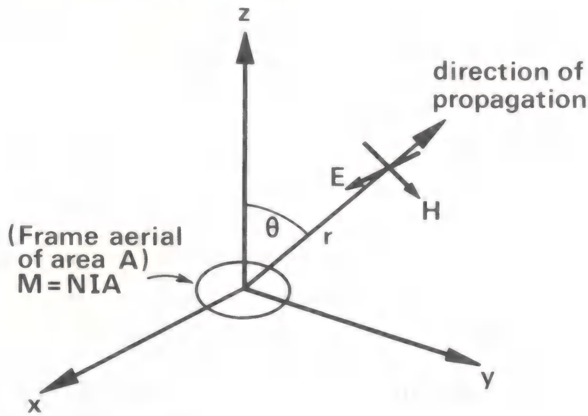
/ ▲▼ / ▲▼ / goto / 0 / 0 / f / RUN / λ
or / ▲▼ / ▲▼ / goto / 1 / 3 / λ

/ RUN / θ / RUN / r / RUN / { Ids / I / X / ds }

/ RUN / H_{pk} / RUN / E_{pk} / X / ▲▼ / rcl / ÷ / P_{pk} / 2 / = / P_{av}

÷	G	00
#	3	01
2	2	02
·	A	03
9	9	04
9	9	05
7	7	06
9	9	07
·	A	08
8	8	09
÷	G	10
=	–	11
stop	0	12
+	E	13
÷	G	14
X	·	15
(	6	16
stop	0	17
sin	7	18
)	6	19
÷	G	20
stop	0	21
X	·	22
stop	0	23
X	·	24
stop	0	25
sto	2	26
#	3	27
3	3	28
7	7	29
6	6	30
·	A	31
7	7	32
3	3	33
=	–	34
stop	0	35

RADIATION FROM LOOP (OR FERRITE) ANTENNA



$$H = NIA \frac{\pi}{\lambda^2 r} \sin\theta \sin\left(\omega t - \frac{2\pi r}{\lambda}\right)$$

$$E = Z_i H$$

$$P = EH$$

$$P_{av} = \frac{E_{pk} H_{pk}}{2}$$

For ferrite, replace NIA by NIA μ_{eff}

Additional formulae:

Radiation resistance:

$$R_r = \frac{16\pi^3}{3} Z_i \left(\frac{NA}{\lambda^2}\right)^2 = 62298{\cdot}7 \left(\frac{NA}{\lambda^2}\right)^2$$

Total power radiated:

$$P_r = I_{rms}^2 R_r = \frac{V_{rms}^2}{R_r} = \frac{I^2 R_r}{2}$$

X	·	00
÷	G	01
X	·	02
stop	0	03
X	·	04
sto	2	05
#	3	06
3	3	07
·	A	08
1	1	09
4	4	10
1	1	11
6	6	12
X	·	13
(	6	14
stop	0	15
sin	7	16
)	6	17
X	·	18
stop	0	19
X	·	20
stop	0	21
=	–	22
stop	0	23
rcl	5	24
X	·	25
X	·	26
#	3	27
6	6	28
2	2	29
2	2	30
9	9	31
9	9	32
=	–	33
=	–	34
stop	0	35

Execution:

$$\lambda \,/\, \mathrm{RUN} \,/ \left\{ \begin{array}{l} \mathrm{NA} \\ \mathrm{N} \,/\, \mathrm{X} \,/\, \mathrm{A} \\ \mathrm{N} \,/\, \mathrm{X} \,/\, 3{\cdot}14159 \,/\, \mathrm{X} \,/\, \blacktriangle\blacktriangledown \,/\, (\,/\, \mathrm{R} \,/\, \mathrm{X} \,/\, \blacktriangle\blacktriangledown \,/\,) \\ \mathrm{N} \,/\, \mathrm{X} \,/\, \ell \,/\, \mathrm{X} \,/\, \mathrm{b} \\ \mathrm{etc.} \end{array} \right\}$$

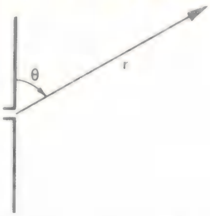
$$\left(/ \, \mathrm{X} \, / \, \mu_{\mathrm{eff}} \right)^{*} / \, \mathrm{RUN} \, / \, \theta \, / \, \mathrm{RUN} \, / \, \mathrm{r} \, / \, \mathrm{RUN} \, / \, \mathrm{I} \, / \, \mathrm{RUN} \, / \, H_{pk}$$

$$\left\{ \begin{array}{l} / \, \mathrm{X} \, / \, 376{\cdot}73 \, / = / \, E_{pk} \\ / \, \mathrm{X} \, / \, \mathrm{X} \, / \, 377 \, / = / \, P_{pk} \\ / \, \mathrm{X} \, / \, \mathrm{X} \, / \, 188{\cdot}365 \, / = / \, P_{av} \end{array} \right\} \quad / \, \mathrm{RUN} \, / \, R_r$$

* omit these two terms for air-cored loop.

Note: Not applicable to near-field radiation pattern, $r < 10R$ where R = radius of loop.

RADIATION FROM HALF–WAVE DIPOLE



$$H = \frac{I}{2\pi r}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\quad \sin\left(\omega t - \frac{2\pi r}{\lambda}\right)$$

$$E = Z_i H$$

$$P = HE \qquad P_{av} = \frac{H_{pk}\,E_{pk}}{2} \qquad Z_i \triangleq 377\Omega$$

Additional formulae:

Radiation resistance:

$$R_r = \frac{\mu_o c}{4}\left(\ln 2\pi y + \int_{2\pi}^{\infty} \frac{\cos y}{y}\,dy\right) \triangleq 72{\cdot}9\Omega$$

Power outputs:

$$P_r = \frac{V_{rms}}{R_r} = I_{rms}^2 R_r = \frac{I^2 R_r}{2}$$

(since I = peak current)

Execution:

θ / RUN / r / RUN / I / X / H_{pk} /

$\begin{cases} \text{RUN / } E_{pk} \\ \text{X / RUN / } P_{pk} \text{ / ÷ / 2 / = / } P_{av} \end{cases}$

This also applies to ¼-wave unipole above ground (radiation resistance 36·5Ω)

Range $0{\cdot}16 < \theta \leqslant 1{\cdot}57$

sto	2	00
cos	8	01
X	·	02
#	3	03
1	1	04
·	A	05
5	5	06
7	7	07
0	0	08
8	8	09
=	—	10
cos	8	11
÷	G	12
(	6	13
rcl	5	14
sin	7	15
)	6	16
÷	G	17
#	3	18
6	6	19
·	A	20
2	2	21
8	8	22
3	3	23
1	1	24
9	9	25
÷	G	26
stop	0	27
X	·	28
stop	0	29
#	3	30
3	3	31
7	7	32
7	7	33
=	—	34
stop	0	35

FOURIER ANALYSIS

The Fourier series expansion of the function $f(\omega t)$ is:

$$f(\omega t) = \frac{a_o}{2} + \sum_{k=1}^{\infty} (a_k \cos k\omega t + b_k \sin k\omega t)$$

$$\text{where } a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\omega t) \cos k\omega t \, d(\omega t),$$

$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\omega t) \sin \omega t \, d(\omega t)$$

If $e(\omega t)$ is a periodic voltage of amplitude E_{pk}, its Fourier series is:

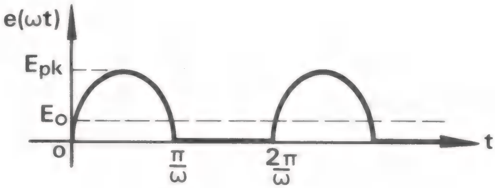
$$e(\omega t) = E_o + \sum_{k=1}^{\infty} E_k \cos (k\omega t + \phi_k) = E_{pk} f(\omega t)$$

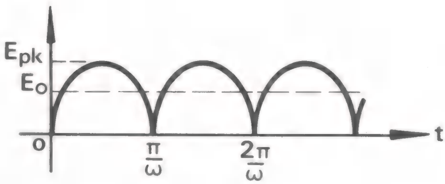
$$\text{where } E_o = \frac{a_o}{2} E_{pk}, \quad E_k = \sqrt{a_k^2 + b_k^2 E_{pk}}$$

The coefficients can be formed by numerical integration for non-analysis waveforms.

FOURIER ANALYSIS

Half-wave rectified and full-wave rectified sine wave





Half-wave:

$$e(\omega t) = \frac{1}{\pi} E_{pk} + \frac{E_{pk} \sin \omega t}{2} - \frac{E_{pk}}{\pi} \times$$

$$\sum_{n=1}^{\infty} \frac{1}{(4n^2 - 1)\pi} \cos 2n\,\omega t$$

Full-wave:

$$e(\omega t) = \frac{2}{\pi} E_{pk} - \frac{2}{\pi} E_{pk} \sum_{n=1}^{\infty} \frac{1}{(4n^2 - 1)\pi} \cos 2n\,\omega t$$

÷	G	00
#	3	01
2	2	02
=	–	03
stop	0	04
÷	G	05
#	3	06
1	1	07
·	A	08
5	5	09
7	7	10
0	0	11
7	7	12
9	9	13
6	6	14
3	3	15
=	–	16
sto	2	17
(	6	18
stop	0	19
+	E	20
×	·	21
–	F	22
#	3	23
1	1	24
÷	G	25
)	6	26
×	·	27
rcl	5	28
=	–	29
▼	A	30
goto	2	31
1	1	32
8	8	33
		34
		35

Half-wave:

$$E_o = \frac{1}{\pi} E_{pk} \qquad E_1 = \frac{E_{pk}}{2}$$

$$E_{2n} = \frac{E_{pk}}{(4n^2 - 1)\pi} \qquad E_{2n+1} = 0$$

Full-wave:

$$E_o = \frac{2}{\pi} E_{pk} \qquad E_1 = 0$$

$$E_{2n} = \frac{2E_{pk}}{(4n^2 - 1)\pi} \qquad E_{2n+1} = 0$$

Execution:

Half-wave:

E_{pk} / RUN / E_1 / RUN / E_o / 1 / RUN / E_2 / 2 / RUN / E_4 / · · ·

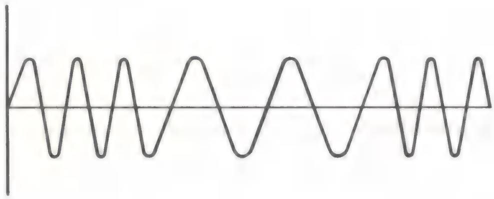
Before re-execution: ▲▼ / ▲▼ / goto / 0 / 0

Full-wave:

▲▼ / ▲▼ / goto / 0 / 5 / E_{pk} / RUN / E_o / 1 / RUN / E_2 / 2 / RUN / E_4 / · · ·

FOURIER ANALYSIS

Frequency modulated wave
(iterative computation of Bessel functions)



Where m = modulation index

$$e(\omega t) = E_{pk} \cos(\omega_c + m \cos \omega_s)t$$

$$= E_{pk} J_o(m) \cos \omega_c t +$$
$$J_1(m)\ [\sin(\omega_c - \omega_s)t - \sin(\omega_c + \omega_s)t] -$$
$$J_2(m)\ [\cos(\omega_c - 2\omega_s)t + \cos(\omega_c + 2\omega_s)t] -$$
$$J_3(m)\ [\sin(\omega_c - 3\omega_s)t - \sin(\omega_c + 3\omega_s)t] +$$
$$J_4(m)\ [\cos(\omega_c + 4\omega_s)t + \cos(\omega_c - 4\omega_s)t] + \cdots$$

$$\text{where } J_n(m) = \left(\frac{m}{2}\right)^n \sum_{r=0}^{\infty} \frac{(-1)^r}{r!(n+r)!}\left(\frac{m}{2}\right)^{2r}$$

$$= \frac{1}{n!}\left(\frac{m}{2}\right)^n \sum_{r=0}^{\infty} \frac{(-1)^r n!}{r!(n+r)!}\left(\frac{m}{2}\right)^{2r}$$

$$= \frac{1}{n!}\left(\frac{m}{2}\right)^n \lim_{k \to \infty} S_k$$

(where S_k is the sum of the series to k terms)

÷	G	00
#	3	01
2	2	02
=	—	03
ln	4	04
X	·	05
stop	0	06
=	—	07
▼	A	08
e^x	4	09
sto	2	10
÷	G	11
stop	0	12
÷	G	13
(	6	14
stop	0	15
+	E	16
+	E	17
)	6	18
X	·	19
(	6	20
stop	0	21
X	·	22
)	6	23
—	F	24
+	E	25
▼	A	26
MEx	5	27
=	—	28
stop	0	29
▼	A	30
MEx	5	31
▼	A	32
goto	2	33
1	1	34
1	1	35

Execution:

▲▼ / ▲▼ / goto / 0 / 0 / m / RUN / n / RUN / 1 / RUN / n / + / 1 / RUN / m / RUN / S_1 / RUN / 2 / RUN / n / + / 2 / RUN / m / RUN / S_2 · · ·

· · · / RUN / r / RUN / n / + / r / RUN / m / RUN / S_r

(Continue until S_r is sufficiently close to S_{r-1} to have converged to required accuracy.)

Post execution:

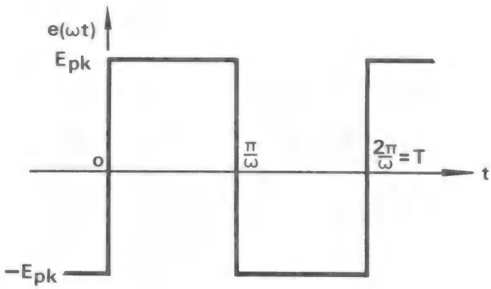
/ ÷ / n! / = / $J_n(m)$

or

/ ÷ / n / ÷ / n – 1 / ÷ / n – 2 / ÷ / · · · / ÷ / 2 / = / $J_n(m)$

FOURIER ANALYSIS

Square wave



$$e(\omega t) = E_{pk} \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi} \sin(2n-1)\omega t$$

i.e. $E_k = 0$ if $k = 2n$

$$= \frac{4E_{pk}}{(2n-1)\pi} \text{ if } k = 2n-1$$

Execution:

RUN / E_{pk} / RUN / E_1 / RUN / E_3 / RUN / · · · / RUN / E_{2n-1} / · · ·

If E_{pk} is not entered, the relative amplitude will be given.

Check:

/ ▲▼ / rcl / recovers the current value of (2n − 1). Clear before running again.

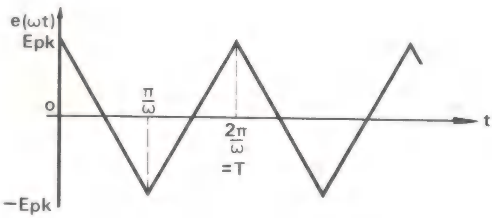
Before re-execution:

▲▼ / ▲▼ / goto / 0 / 0

#	3	00
1	1	01
=	—	02
sto	2	03
stop	0	04
X	·	05
#	3	06
1	1	07
·	A	08
2	2	09
7	7	10
3	3	11
2	2	12
3	3	13
9	9	14
5	5	15
=	—	16
stop	0	17
X	·	18
rcl	5	19
÷	G	20
(	6	21
rcl	5	22
+	E	23
#	3	24
2	2	25
=	—	26
sto	2	27
)	6	28
=	—	29
▼	A	30
goto	2	31
1	1	32
7	7	33
		34
		35

FOURIER ANALYSIS

Triangular wave



$$e(\omega t) = E_{pk} \sum_{n=1}^{\infty} \frac{8}{(2n-1)^2\pi^2} \cos(2n-1)\omega t$$

$$E_k = E_{pk} \frac{8}{(2n-1)^2\pi^2} \quad \text{if } k = 2n-1$$

$$= 0 \quad \text{if } k = 2n$$

Execution:

RUN / E_{pk} / RUN / E_1 / RUN / E_3 / · · ·

Post-execution at any stage:

▲▼ / rcl / $(2n-1)$ / C/CE / (E_{2n-1})

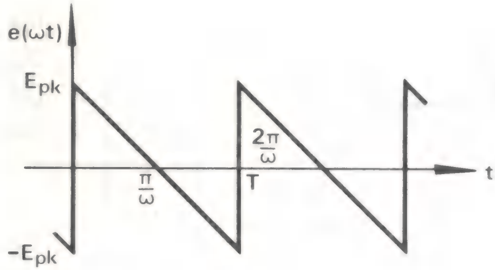
Before execution:

▲▼ / ▲▼ / goto / 0 / 0

#	3	00
1	1	01
=	—	02
sto	2	03
stop	0	04
÷	G	05
#	3	06
1	1	07
·	A	08
2	2	09
3	3	10
3	3	11
7	7	12
=	—	13
stop	0	14
X	·	15
(	6	16
rcl	5	17
X	·	18
)	6	19
÷	G	20
(	6	21
rcl	5	22
+	E	23
#	3	24
2	2	25
X	·	26
sto	2	27
)	6	28
=	—	29
▼	A	30
goto	2	31
1	1	32
4	4	33
		34
		35

FOURIER ANALYSIS

Sawtooth wave



$$e(\omega t) = E_{pk} \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin n\omega t$$

$E_o = 0 \qquad E_n = \dfrac{2}{n\pi}$

Execution:

RUN / E_{pk} / RUN / E_1 / RUN / E_2 / RUN / · · · /
RUN / E_n · · ·

At any stage, current harmonic order n can be recalled:

/ ▲▼ / rcl / (n) / C/CE / (E_n) / RUN / · · ·

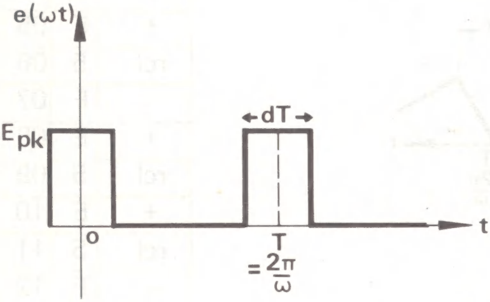
Before re-execution:

▲▼ / ▲▼ / goto / 0 / 0

#	3	00
1	1	01
=	—	02
sto	2	03
stop	0	04
÷	G	05
#	3	06
1	1	07
·	A	08
5	5	09
7	7	10
0	0	11
7	7	12
9	9	13
6	6	14
3	3	15
=	—	16
stop	0	17
X	·	18
rcl	5	19
÷	G	20
(	6	21
rcl	5	22
+	E	23
#	3	24
1	1	25
=	—	26
sto	2	27
)	6	28
=	—	29
▼	A	30
goto	2	31
1	1	32
7	7	33
		34
		35

FOURIER ANALYSIS

Rectangular pulse train of duty cycle d



$$e(\omega t) = d\,E_{pk} + E_{pk} \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin n\pi d \cos n\omega t$$

$$E_o = dE_{pk} \qquad E_n = \frac{2}{n\pi} \sin n\pi d\ E_{pk}$$

Pre-execution:

1·5707963 / ▲▼ / sto / ▲▼ / ▲▼ / goto / 0 / 0 / d / X / E_{pk} / = / E_o

Execution:

n / RUN / d / RUN / E_{pk} / RUN / E_n

n = 1, 2, 3, · · ·

Notes:

Ignore negative signs in results

If E appears after second / RUN / :

(i) Note result r

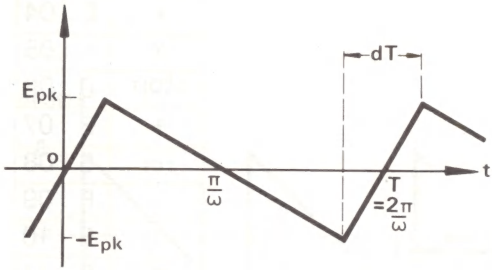
(ii) Press / 3 / C/CE /

(iii) Enter r / X / E_{pk} / RUN / E_n

X	·	00
rcl	5	01
÷	G	02
(	6	03
+	E	04
X	·	05
stop	0	06
+	E	07
rcl	5	08
—	F	09
+	E	10
rcl	5	11
+	E	12
rcl	5	13
—	F	14
▼	A	15
gin	1	16
0	0	17
9	9	18
+	E	19
rcl	5	20
=	—	21
sin	7	22
)	6	23
÷	G	24
X	·	25
stop	0	26
=	—	27
stop	0	28
▼	A	29
goto	2	30
0	0	31
0	0	32
		33
		34
		35

FOURIER ANALYSIS

Asymmetrical triangular wave



$$e(\omega t) = E_{pk} \sum_{n=1}^{\infty} \frac{2}{n^2\pi^2 d(1-d)} \sin n\pi d \sin n\omega t$$

$$E_o = 0 \qquad E_n = \frac{2}{n^2\pi^2 d(1-d)} \sin n\pi d\ E_{pk}$$

Pre-execution:

▲▼ / ▲▼ / goto / 0 / 0 / 1·5707963 / ▲▼ / sto /

Execution:

▲▼ / rcl / X / n / X / d / RUN / d / RUN / E_{pk} / = / E_n

Notes:

Ignore negative signs in results.

If E appears after first / RUN / :

(i) Note the result r

(ii) Press / 3 / C/CE /

(iii) Enter r / X / ▲▼ / (/

(iv) Continue with execution:
d / RUN / E_{pk} / = / E_n

X	·	00
÷	G	01
(	6	02
$\sqrt{x}$	1	03
+	E	04
+	E	05
rcl	5	06
—	F	07
+	E	08
rcl	5	09
+	E	10
rcl	5	11
—	F	12
▼	A	13
gin	1	14
0	0	15
7	7	16
+	E	17
rcl	5	18
=	—	19
sin	7	20
)	6	21
+	E	22
X	·	23
(	6	24
stop	0	25
÷	G	26
—	F	27
#	3	28
1	1	29
=	—	30
)	6	31
÷	G	32
X	·	33
stop	0	34
=	—	35

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